

# Efficient Algorithms for Renewable Energy Allocation to Delay Tolerant Consumers

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**Abstract**—We investigate the problem of allocating energy from renewable sources to flexible consumers in electricity markets. We assume there is a renewable energy supplier that provides energy according to a time-varying (and possibly unpredictable) supply process. The plant must serve consumers within a specified delay window, and incurs a cost of drawing energy from other (possibly non-renewable) sources if its own supply is not sufficient to meet the deadlines. We formulate two stochastic optimization problems: The first seeks to minimize the time average cost of using the other sources (and hence strives for the most efficient utilization of the renewable source). The second allows the renewable source to dynamically set a price for its service, and seeks to maximize the resulting time average profit. These problems are solved via the Lyapunov optimization technique. Our resulting algorithms do not require knowledge of the statistics of the time-varying supply and demand processes and are robust to arbitrary sample path variations.

## I. INTRODUCTION

The highly variable and unpredictable nature of some renewable energy sources (such as wind and solar) has been a major obstacle to their integration. For example, a recent study conducted by Enernex for wind power integration in Minnesota [4] indicates that the variability and day-ahead forecast errors will result in an additional \$2.11 – \$4.41 (for 15% and 25% penetration) per MWh of delivered wind power. Along the same lines, the CAISO report [5] predicted that ten minute real-time energy prices could increase substantially due to wind forecasting errors and identified day-ahead and same-day forecasts and modeling as important tasks for integration of renewable resources.

The necessity to offset variability by stand-by generators and system backup investments substantially increases the cost of renewables. One approach that can mitigate this problem is to couple this supply variability to demand side flexibility [1], [2], [3]. The renewable power suppliers could sell their energy at a lower price to consumers that are willing to wait in a queue, given that it will be served to them within a pre-agreed deadline. This essentially allows a lower price of renewable energy to consumers willing to provide this extra time flexibility. The renewable power supplier can now use this flexibility to deliver the energy when it is available.<sup>1</sup> The supplier will sometimes, hopefully rarely, be in a situation when a prior deadline commitment cannot be matched and will have to purchase the extra energy from the energy spot market (or

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<sup>1</sup>Note that in this paper we assume no energy storage although it can be naturally incorporated into our framework.

maintain a costly system backup). Papavasiliou and Oren [3] introduced this problem and proposed an exact backward dynamic programming algorithm and an efficient approximate dynamic programming algorithm for the scheduling decisions of the renewable energy supplier.

In this paper we build a similar model and utilize the technique of Lyapunov optimization initially developed in [10][11][12] for dynamic control of queueing systems for wireless networks. We show that the queuing model naturally fits in the renewable supplier scheduling problem and present a simple energy allocation algorithm that does not require prior statistical information and is provably close to optimal. The proposed framework can be extended to include pricing, multiple queues (with different deadlines) and different objective functions, building on the general results from [10]. We finally evaluate the proposed algorithm on actual CAISO spot market and wind energy production data and show substantial reduction to the operating costs for the renewable supplier compared to a simple greedy algorithm.

In particular, we consider a single renewable energy plant that operates in discrete time with unit timeslots  $t \in \{0, 1, 2, \dots\}$ , and provides  $s(t)$  units of energy on each slot  $t$ . The  $s(t)$  process corresponds to the renewable supply and is assumed to be time varying and unpredictable. Since we assume no storage, the energy  $s(t)$  must either be used or wasted. Demands for this energy arrive randomly according to a process  $a(t)$  (being the amount of energy that is requested on slot  $t$ ). We assume that consumers requesting energy are flexible, and can tolerate their energy requests being satisfied with some delay. The requests are thus stored in a queue. Every slot  $t$ , we use all of our supply  $s(t)$  to serve the requests in the queue in a First-In-First-Out (FIFO) manner. However, this may not be enough to meet all of the requests within a timely manner, and hence we also decide to purchase an amount of energy  $x(t)$  from an outside (possibly non-renewable) plant. Letting  $Q(t)$  represent the total energy requests in our queue on slot  $t$ , we have the following update equation:

$$Q(t+1) = \max[Q(t) - s(t) - x(t), 0] + a(t) \quad (1)$$

The value  $x(t)$  is a control decision variable, and incurs a cost  $x(t)\gamma(t)$  on slot  $t$ , where  $\gamma(t)$  is a process that specifies the per-unit-cost of using the outside energy supply on slot  $t$ . The value of  $\gamma(t)$  can represent a current market price for guaranteed energy services from (possibly non-renewable) sources. As such, the decision to use  $x(t)$  units of energy on slot  $t$  means the outside source agrees to provide this much energy at time  $t+K$  for some fixed (and small) integer  $K \geq 0$ , for the price  $x(t)\gamma(t)$ . Without loss of generality, we assume throughout that  $K = 0$ , so that the energy request is removed from our queue on the same slot in which we decide to use

the outside source. In the actual implementation, requests that are served from the outside source can be removed from the primary queue  $Q(t)$  but must still wait an additional  $K$  slots.

We first look at the problem of choosing  $x(t)$  to stabilize our queue  $Q(t)$  while minimizing the time average of the cost  $x(t)\gamma(t)$  and also providing a guarantee on the maximum delay  $D_{max}$  spent in the queue. If the future values of supply, demand, and market price values  $(s(t), a(t), \gamma(t))$  were known in advance, one could in principle make  $x(t)$  decisions that minimize total time average cost, possibly choosing  $x(t) = 0$  for all  $t$  if it is possible to meet all demands using only the renewable energy  $s(t)$ . The challenge is to provide an efficient algorithm without knowing the future. To this end, we first assume the vector process  $(s(t), a(t), \gamma(t))$  is i.i.d. over slots but has an unknown probability distribution. Under this assumption, we develop an algorithm, parameterized by a positive value  $V$ , that comes within  $O(1/V)$  of the minimum time average cost required to stabilize the queue, with a worst-case delay guarantee that is  $O(V)$ . The parameter  $V$  can be tuned as desired to provide average cost arbitrarily close to optimal, with a tradeoff in delay. We further show that the same algorithm is provably robust to non-i.i.d. situations, and operates efficiently even for arbitrary sample paths for  $(s(t), a(t), \gamma(t))$ . Finally, we extend the problem to consider pricing decisions at the renewable energy source, so that the requests  $a(t)$  are now influenced by the current prices. In this case, we design a related algorithm that maximizes time average profit.

The Lyapunov optimization technique we use [10][11][12] is related to the primal-dual and fluid-model techniques in [13][14][15][16]. The work in [10][11][12] establishes a general  $[O(1/V), O(V)]$  performance-congestion tradeoff for stochastic network optimization problems with i.i.d. (and more general ergodic) processes. Recent work in [17][18] provides similar results on a sample path basis, without any probabilistic assumptions. We apply these results in our current paper. Further, we extend the theory by introducing a novel virtual queue that turns an average delay constraint of  $O(V)$  (which is achievable with the prior analytical techniques) into a *worst case delay guarantee* that is also  $O(V)$ .

It is useful to distinguish the proposed Lyapunov optimization method that we use in this paper from dynamic programming techniques. Dynamic programming can be used to solve stronger versions of our problem (such as minimizing average cost subject to a delay constraint) see e.g. [3]. However, dynamic programming requires more stringent system modeling assumptions, has a more complex solution that typically requires knowledge of the supply, demand, and market price probabilities, and cannot necessarily adapt if these probabilities change and/or if there are unmodeled correlations in the actual processes. It involves computation of a value function that can be difficult when the state space of the system is large, and suffers from a *curse of dimensionality* when applied to large dimensional systems (such as systems with many queues).

In contrast, Lyapunov optimization is relatively simple to implement, does not need a-priori statistical knowledge, and is robust to non-i.i.d. and non-ergodic behavior. Further, it has no

curse of dimensionality and hence can be applied just as easily in extended formulations that have multiple queues corresponding to multiple customers requesting different deadlines, contrary to dynamic programming [3] which would require exponential complexity in the number of users.

The reason for this efficiency is that Lyapunov optimization relaxes the question that dynamic programming asks: Rather than minimizing time average cost subject to a delay constraint, it seeks to push time average cost towards the more ambitious minimum over all possible algorithms that can stabilize the queue (without regard to the delay constraint). It then specifies an explicit bound on the resulting queue congestion, which depends on the desired proximity to the minimum cost (as defined by the  $[O(1/V), O(V)]$  performance-congestion tradeoff). However, the resulting time average queue congestion (and delay) that is achieved is not necessarily the optimal that could be achieved over all possible algorithms that yield the same time average performance cost.

In the next section, we formulate the basic model under the assumption that the  $(s(t), a(t), \gamma(t))$  vector is i.i.d. over slots, and present the main allocation algorithm. Section III extends to the case when the renewable power source can set a price for its services. These algorithms are provably robust to non-i.i.d. situations and arbitrary sample paths of events, as shown in Section IV. Section V presents an experimental evaluation of our algorithm on a real six-month data set and shows substantial gains over a simple greedy scheduling algorithm.

## II. THE DYNAMIC ALLOCATION ALGORITHM

Suppose that the supply process  $s(t)$ , the request process  $a(t)$ , and the market price process  $\gamma(t)$ , as described in the introduction, form a vector  $(s(t), a(t), \gamma(t))$  that is i.i.d. over slots with some unknown probability distribution. We further assume the values of  $s(t)$ ,  $a(t)$ ,  $\gamma(t)$  are deterministically bounded by finite constants  $s_{max}$ ,  $a_{max}$ ,  $\gamma_{max}$ , so that:

$$0 \leq s(t) \leq s_{max}, 0 \leq a(t) \leq a_{max}, 0 \leq \gamma(t) \leq \gamma_{max} \quad \forall t \quad (2)$$

The queue backlog  $Q(t)$  evolves according to (1). The decision variable  $x(t)$  is chosen every slot  $t$  in reaction to the current  $(s(t), a(t), \gamma(t))$  (and possibly additional queue state information) subject to the constraint  $0 \leq x(t) \leq x_{max}$  for all  $t$ , where  $x_{max}$  is a finite upper bound. We assume that  $x_{max} \geq a_{max}$  so that it is always possible to stabilize the queue  $Q(t)$  (and this can be done with one slot delay if we choose  $x(t) = x_{max}$  for all  $t$ ). Define  $\bar{c}$  as the time average cost incurred by our control policy (assuming temporarily that our policy yields such a well defined limit):

$$\bar{c} \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E} \{ \gamma(\tau)x(\tau) \}$$

We want to find an allocation algorithm that chooses  $x(t)$  over time to solve:

$$\text{Minimize:} \quad \bar{c} \quad (3)$$

$$\text{Subject to:} \quad 1) \quad \bar{Q} < \infty \quad (4)$$

$$2) \quad 0 \leq x(t) \leq x_{max} \quad \forall t \quad (5)$$

where  $\bar{Q}$  is the time average expected queue backlog, defined:

$$\bar{Q} \triangleq \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E} \{ Q(\tau) \}$$

Define  $c^*$  as the infimum time average cost associated with the above problem, considering all possible ways of choosing  $x(t)$  over time. The value of  $c^*$  is an ambitious target because the above problem is defined only in terms of a queue stability constraint and does not impose any additional delay constraint. We shall construct a solution, parameterized by a constant  $V > 0$ , that satisfies the constraints of the above problem and pushes the average cost within  $O(1/V)$  of the optimal value  $c^*$ . Further, we show that our algorithm has the additional property that worst case delay is no more than  $O(V)$ .

### A. The Delay-Aware Virtual Queue

We solve the above problem while also maintaining finite worst case delay using the following novel ‘‘virtual queue’’  $Z(t)$ : Fix a parameter  $\epsilon > 0$ , to be specified later. Define  $Z(0) = 0$ , and define the virtual queue  $Z(t)$  for  $t \in \{0, 1, 2, \dots\}$  according to the following update:

$$Z(t+1) = \max[Z(t) - s(t) - x(t) + \epsilon 1_{\{Q(t)>0\}}, 0] \quad (6)$$

where  $1_{\{Q(t)>0\}}$  is an indicator function that is 1 if  $Q(t) > 0$ , and zero else. The intuition is that  $Z(t)$  has the same service process as  $Q(t)$  (being  $s(t) + x(t)$ ), but now has an arrival process that adds  $\epsilon$  whenever the actual queue backlog is non-empty. This ensures that  $Z(t)$  grows if there are requests in the  $Q(t)$  queue that have not been serviced for a long time. If we can control the system to ensure that the queues  $Q(t)$  and  $Z(t)$  have finite upper bounds, then we can ensure all requests are served with a worst case delay given in the following lemma.<sup>2</sup>

**Lemma 1:** (Worst Case Delay) Suppose the system is controlled so that  $Z(t) \leq Z_{max}$  and  $Q(t) \leq Q_{max}$  for all  $t$ , for some positive constants  $Z_{max}$  and  $Q_{max}$ . Then all requests are fulfilled with a maximum delay of  $D_{max}$  slots, where:

$$D_{max} \triangleq \lceil (Q_{max} + Z_{max})/\epsilon \rceil \quad (7)$$

*Proof:* Consider any slot  $t$  for which  $a(t) > 0$ . We show that the requests  $a(t)$  are fulfilled on or before time  $t + D_{max}$ . Suppose not (we shall reach a contradiction). Then during slots  $\tau \in \{t+1, \dots, t+D_{max}\}$  it must be that  $Q(\tau) > 0$  (else the requests  $a(t)$  would have been served before slot  $\tau$ ). Thus,  $1_{\{Q(\tau)>0\}} = 1$ , and from (6) we have that for all  $\tau \in \{t+1, \dots, t+D_{max}\}$ :

$$Z(\tau+1) \geq Z(\tau) - s(\tau) - x(\tau) + \epsilon$$

Summing the above over  $\tau \in \{t+1, \dots, t+D_{max}\}$  yields:

$$Z(t+D_{max}+1) - Z(t+1) \geq - \sum_{\tau=t+1}^{t+D_{max}} [s(\tau) + x(\tau)] + D_{max} \epsilon$$

Rearranging and using the fact that  $Z(t+1) \geq 0$  and  $Z(t+D_{max}+1) \leq Z_{max}$  yields:

$$\sum_{\tau=t+1}^{t+D_{max}} [s(\tau) + x(\tau)] \geq D_{max} \epsilon - Z_{max} \quad (8)$$

Now note that the requests  $a(t)$  are first available for service at time  $t+1$ , and are part of the backlog  $Q(t+1)$  (see (1)). Because  $Q(t+1) \leq Q_{max}$  and because service is FIFO, these

<sup>2</sup>In the case when requests are served by the outside source with an additional delay  $K > 0$ , then this bound is modified in the actual implementation to  $\lceil (Q_{max} + Z_{max})/\epsilon \rceil + K$ .

requests  $a(t)$  are served on or before time  $t + D_{max}$  whenever there are at least  $Q_{max}$  units of energy served during the interval  $\tau \in \{t+1, \dots, t+D_{max}\}$ . Because we have assumed the requests  $a(t)$  are *not* served by time  $t + D_{max}$ , it must be that  $\sum_{\tau=t+1}^{t+D_{max}} [s(\tau) + x(\tau)] < Q_{max}$ . Using this in (8) yields:

$$Q_{max} > D_{max} \epsilon - Z_{max}$$

This implies that  $D_{max} < (Q_{max} + Z_{max})/\epsilon$ , contradicting the definition of  $D_{max}$  in (7).  $\square$

### B. Lyapunov Optimization

Define  $\Theta(t) \triangleq (Z(t), Q(t))$  as the concatenated vector of the real and virtual queues. As a scalar measure of the congestion in both the  $Z(t)$  and  $Q(t)$  queues, we define the following Lyapunov function:  $L(\Theta(t)) \triangleq \frac{1}{2}[Z(t)^2 + Q(t)^2]$ . Define the conditional 1-slot Lyapunov drift as follows:

$$\Delta(\Theta(t)) \triangleq \mathbb{E} \{L(\Theta(t+1)) - L(\Theta(t))|\Theta(t)\} \quad (9)$$

Following the drift-plus-penalty framework of [10][11][12], our control algorithm is designed to observe the current queue states  $Z(t)$ ,  $Q(t)$  and the current  $(s(t), a(t), \gamma(t))$  vector, and to make a decision  $x(t)$  (where  $0 \leq x(t) \leq x_{max}$ ) to minimize a bound on the following expression every slot  $t$ :

$$\Delta(\Theta(t)) + V \mathbb{E} \{\gamma(t)x(t)|\Theta(t)\}$$

where  $V$  is a positive parameter that will be useful to affect a performance-delay tradeoff. We first compute a bound on the above drift-plus-penalty expression.

**Lemma 2:** (Drift Bound) For any control policy that satisfies  $0 \leq x(t) \leq x_{max}$  for all  $t$ , the drift-plus-penalty expression for all slots  $t$  satisfies:

$$\begin{aligned} \Delta(\Theta(t)) + V \mathbb{E} \{\gamma(t)x(t)|\Theta(t)\} &\leq B + V \mathbb{E} \{\gamma(t)x(t)|\Theta(t)\} \\ &\quad + Q(t) \mathbb{E} \{a(t) - s(t) - x(t)|\Theta(t)\} \\ &\quad + Z(t) \mathbb{E} \{\epsilon - s(t) - x(t)|\Theta(t)\} \end{aligned} \quad (10)$$

where the constant  $B$  is defined:

$$B \triangleq \frac{(s_{max} + x_{max})^2 + a_{max}^2}{2} + \frac{\max[\epsilon^2, (s_{max} + x_{max})^2]}{2} \quad (11)$$

*Proof:* See Appendix A.  $\square$

### C. The Dynamic Algorithm

Minimizing the right-hand-side of the drift-plus-penalty bound (10) every slot  $t$  leads to the following dynamic algorithm: Every slot  $t$ , observe  $Z(t)$ ,  $Q(t)$ ,  $(s(t), a(t), \gamma(t))$ , and choose  $x(t)$  according to the following optimization:

$$\begin{aligned} \text{Minimize: } & x(t)[V\gamma(t) - Q(t) - Z(t)] \\ \text{Subject to: } & 0 \leq x(t) \leq x_{max} \end{aligned}$$

Then update the actual and virtual queues  $Q(t)$  and  $Z(t)$  by (1) and (6). The above minimization for the  $x(t)$  decision reduces to the following simple threshold rule:

$$x(t) = \begin{cases} 0 & \text{if } Q(t) + Z(t) \leq V\gamma(t) \\ x_{max} & \text{otherwise} \end{cases} \quad (12)$$

The above  $x(t)$  value drives the queueing updates (1) and (6). However, note by the  $\max[\cdot, 0]$  structure of the  $Q(t)$  update in (1) that we may not need to purchase the full  $x(t)$  units of energy from the outside plant on slot  $t$ . Indeed, define  $\tilde{x}(t)$  as the *actual* amount purchased from the plant, given by:

$$\tilde{x}(t) \triangleq \begin{cases} x(t) & \text{if } Q(t) - s(t) \geq x(t) \\ \min[Q(t) - s(t), 0] & \text{otherwise} \end{cases} \quad (13)$$

Then we have  $\tilde{x}(t) \leq x(t)$  for all  $t$ .

**Theorem 1:** (Performance Analysis) Suppose  $x_{max} \geq \max[a_{max}, \epsilon]$ . If  $Q(0) = Z(0) = 0$ , and if the above dynamic algorithm is implemented with any fixed  $\epsilon \geq 0$  and  $V > 0$  for all  $t \in \{0, 1, 2, \dots\}$ , then:

a) The queues  $Q(t)$  and  $Z(t)$  are deterministically bounded by  $Q_{max}$  and  $Z_{max}$  every slot  $t$ , where:

$$Q_{max} \triangleq V\gamma_{max} + a_{max}, \quad Z_{max} \triangleq V\gamma_{max} + \epsilon \quad (14)$$

b) The worst case delay of any request is:

$$D_{max} = \lceil (2V\gamma_{max} + a_{max} + \epsilon)/\epsilon \rceil \quad (15)$$

c) If the vector  $(s(t), a(t), \gamma(t))$  is i.i.d. over slots, and if the  $\epsilon$  parameter is chosen to satisfy  $\epsilon \leq \max[\mathbb{E}\{a(t)\}, \mathbb{E}\{s(t)\}]$ , then for all slots  $t > 0$  the time average cost satisfies:

$$\frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\gamma(\tau)\tilde{x}(\tau)\} \leq \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\gamma(\tau)x(\tau)\} \leq c^* + B/V$$

where  $B$  is defined in (11).

The above theorem demonstrates the  $[O(1/V), O(V)]$  cost-delay tradeoff, where time average cost is within  $B/V$  of the minimum possible time average cost  $c^*$  required for queue stability, and worst case delay is proportional to  $V/\epsilon$ . To obtain the smallest  $D_{max}$ , the  $\epsilon$  value should be chosen as large as possible while still maintaining  $\epsilon \leq \max[\mathbb{E}\{a(t)\}, \mathbb{E}\{s(t)\}]$ . We can choose  $\epsilon = \mathbb{E}\{a(t)\}$  if this expectation is known. Using  $\epsilon = 0$  preserves parts (a) and (c) but does not give a finite  $D_{max}$ . More discussion of the  $\epsilon = 0$  case is given in Section V.

#### D. Proof of Theorem 1

**Proof:** (Theorem 1 part (a)) We first show that  $Q(t) \leq V\gamma_{max} + a_{max}$  for all  $t$ . This is clearly true for  $t = 0$  (because  $Q(0) = 0$ ). Suppose it holds for slot  $t$ . We show it also holds for slot  $t + 1$ . Consider the case when  $Q(t) \leq V\gamma_{max}$ . Then  $Q(t+1) \leq V\gamma_{max} + a_{max}$ , because the queue can increase by at most  $a_{max}$  on any slot (see dynamics (1)). Thus, the result holds in this case.

Now consider the opposite case when  $V\gamma_{max} < Q(t) \leq V\gamma_{max} + a_{max}$ . In this case, we have:

$$Q(t) + Z(t) \geq Q(t) > V\gamma_{max} \geq V\gamma(t)$$

and hence the algorithm will choose  $x(t) = x_{max}$  according to (12). If  $Q(t) - x_{max} - s(t) > 0$ , then on slot  $t$  we serve at least  $x_{max}$  units of data. Because arrivals  $a(t)$  are at most  $a_{max}$  (and  $a_{max} \leq x_{max}$ ), the queue cannot increase on the next slot and so  $Q(t+1) \leq Q(t) \leq V\gamma_{max} + a_{max}$ . Finally, if  $Q(t) - x_{max} - s(t) \leq 0$ , then by (1) we have  $Q(t+1) = a(t) \leq a_{max}$ , again being less than or equal to  $V\gamma_{max} + a_{max}$ .

Therefore,  $Q(t) \leq V\gamma_{max} + a_{max}$  for all  $t$ . The proof that  $Z(t) \leq V\gamma_{max} + \epsilon$  for all  $t$  is similar and omitted for brevity.  $\square$

**Proof:** (Theorem 1 part (b)) This follows immediately from Lemma 1 together with part (a).  $\square$

The proof of Theorem 1 part (c) requires a preliminary lemma from [11]. To introduce the lemma, define a  $(s, a, \gamma)$ -only policy to be one that observes the current vector  $(s(t), a(t), \gamma(t))$  and makes a stationary and randomized decision  $x^*(t)$  based purely on this vector (and independent of the queue backlog or past system history), subject to the constraint  $0 \leq x^*(t) \leq x_{max}$ .

**Lemma 3:** (Characterizing Optimality [11]) If the vector  $(s(t), a(t), \gamma(t))$  is i.i.d. over slots, then there exists a  $(s, a, \gamma)$ -only policy  $x^*(t)$  that satisfies:

$$\mathbb{E}\{\gamma(t)x^*(t)\} = c^* \quad (16)$$

$$\mathbb{E}\{s(t) + x^*(t)\} \geq \mathbb{E}\{a(t)\} \quad (17)$$

where  $c^*$  is the infimum time average cost in the stochastic optimization problem (3)-(5), and the above expectations are with respect to the stationary distribution of the vector  $(s(t), a(t), \gamma(t))$  and the possibly randomized action  $x^*(t)$  made in reaction to this vector.

**Proof:** (Lemma 3) This follows as a special case of results in [11].  $\square$

**Proof:** (Theorem 1 part (c)) We have assumed that  $\epsilon \leq \max[\mathbb{E}\{a(t)\}, \mathbb{E}\{s(t)\}]$ . We first prove the result for the case when  $\epsilon \leq \mathbb{E}\{a(t)\}$ . On every slot  $t$ , the dynamic choice of  $x(t)$  in (12) minimizes the right-hand-side of the drift bound (10) (given the observed queue sizes  $\Theta(t) \triangleq (Q(t), Z(t))$ ), over all alternative choices  $x^*(t)$  that satisfy the required bounds  $0 \leq x^*(t) \leq x_{max}$  (including randomized choices for  $x^*(t)$ ). Thus, by (10) we have:

$$\begin{aligned} \Delta(\Theta(t)) + V\mathbb{E}\{\gamma(t)x(t)|\Theta(t)\} &\leq B + V\mathbb{E}\{\gamma(t)x^*(t)|\Theta(t)\} \\ &\quad + Q(t)\mathbb{E}\{a(t) - s(t) - x^*(t)|\Theta(t)\} \\ &\quad + Z(t)\mathbb{E}\{\epsilon - s(t) - x^*(t)|\Theta(t)\} \end{aligned}$$

where  $x^*(t)$  is any alternative (possibly randomized) decision. Plugging the  $(s, a, \gamma)$ -only policy  $x^*(t)$  from (16)-(17) (known to exist by Lemma 3) into the right hand side of the above inequality for slot  $t$ , and noting that this policy makes decisions independent of queue backlog, yields:

$$\Delta(\Theta(t)) + V\mathbb{E}\{\gamma(t)x(t)|\Theta(t)\} \leq B + Vc^* \quad (18)$$

where we have used the fact that:

$$\begin{aligned} \mathbb{E}\{a(t) - s(t) - x^*(t)|\Theta(t)\} \\ = \mathbb{E}\{a(t) - s(t) - x^*(t)\} \leq 0 \end{aligned} \quad (19)$$

$$\begin{aligned} \mathbb{E}\{\epsilon - s(t) - x^*(t)|\Theta(t)\} \\ = \mathbb{E}\{\epsilon - s(t) - x^*(t)\} \leq 0 \end{aligned} \quad (20)$$

where (19) follows from (17) and the fact that the  $(s, a, \gamma)$ -only policy  $x^*(t)$  is i.i.d. over slots and hence independent of queue backlog  $\Theta(t)$ , and (20) follows from (17) together with the fact that  $\mathbb{E}\{a(t)\} \geq \epsilon$ .

Taking expectations of (18) and using the law of iterated expectations with the definition of  $\Delta(\Theta(t))$  in (9) yields:

$$\mathbb{E}\{L(\Theta(t+1))\} - \mathbb{E}\{L(\Theta(t))\} + V\mathbb{E}\{\gamma(t)x(t)\} \leq B + Vc^*$$

The above holds for all slots  $t > 0$ . Summing over  $t \in \{0, 1, \dots, M-1\}$  for some positive integer  $M$  yields:

$$\mathbb{E}\{L(\Theta(M))\} - \mathbb{E}\{L(\Theta(0))\} + \sum_{t=0}^{M-1} V\mathbb{E}\{\gamma(t)x(t)\} \leq BM + VMc^*$$

Using the fact that  $L(\Theta(0)) = 0$  (because all queues are initially empty), and that  $L(\Theta(M)) \geq 0$  (because the Lyapunov function is non-negative) and dividing by  $VM$  yields:

$$\frac{1}{M} \sum_{t=0}^{M-1} \mathbb{E}\{\gamma(t)x(t)\} \leq c^* + B/V$$

This holds for all  $M > 0$ , proving the result for the case when  $\epsilon \leq \mathbb{E}\{a(t)\}$ .

We have only used the assumption that  $\epsilon \leq \mathbb{E}\{a(t)\}$  to ensure the inequality (20) holds. If  $\epsilon \leq \mathbb{E}\{s(t)\}$ , then clearly (20) holds, regardless of the value of  $\mathbb{E}\{a(t)\}$ . Thus, the result holds whenever  $\epsilon \leq \max[\mathbb{E}\{a(t)\}, \mathbb{E}\{s(t)\}]$ , proving the theorem.  $\square$

### III. PRICING FOR MAXIMUM PROFIT

We now extend the problem to consider pricing decisions. Instead of a process  $a(t)$  that represents requests arriving at slot  $t$ , we define a process  $y(t)$ , called the *demand state* on slot  $t$ . The demand state captures any properties of the demand that may affect requests for the renewable energy source in reaction to the price advertised on slot  $t$ . A simple example is when  $y(t)$  can take one of two possible values, such as HIGH and LOW, representing different demand conditions (such as during peak times or non-peak times for requesting energy). Another example is when  $y(t)$  represents the number of consumers willing to purchase renewable energy on slot  $t$ . We assume the demand state  $y(t)$  is known at the beginning of each slot  $t$  (we show a particular case where  $y(t)$  does not need to be known after our algorithm is stated).

Every slot  $t$ , in addition to choosing the amount of energy  $x(t)$  purchased from outside sources, the renewable energy plant makes a binary decision  $b(t) \in \{0, 1\}$ , where  $b(t) = 1$  represents a willingness to accept new requests on slot  $t$ , and  $b(t) = 0$  means no requests will be accepted. If  $b(t) = 1$  is chosen, the plant also chooses a per-unit-energy price  $p(t)$  within an interval  $0 \leq p(t) \leq p_{max}$ , where  $p_{max}$  is a pre-established maximum price. The arriving requests  $a(t)$  are then influenced by the current price  $p(t)$ , the current market price  $\gamma(t)$ , and the current demand state  $y(t)$ , according to a general *demand function*  $F(p, y, \gamma)$ . Specifically, the values of  $a(t)$  are assumed to be conditionally i.i.d. over all slots with the same  $p(t)$ ,  $y(t)$ ,  $\gamma(t)$ , and satisfy:

$$\mathbb{E}\{a(t)|p(t), y(t), \gamma(t), b(t) = 1\} = F(p(t), y(t), \gamma(t))$$

We assume the function  $F(p, y, \gamma)$  is continuous in  $p$  for each given  $y$  and  $\gamma$ .<sup>3</sup> We further assume the arrivals  $a(t)$  continue to

<sup>3</sup>This continuity is only used to ensure the resulting min-drift decision has a well defined minimizing price  $p(t)$  every slot.

be worst-case bounded by  $a_{max}$ , regardless of  $p(t)$ ,  $y(t)$ ,  $\gamma(t)$ . The queue iteration  $Q(t)$  still operates according to (1), with the understanding that  $a(t)$  is now influenced by the pricing decisions. Let  $\phi(t)$  represent the instantaneous profit earned on slot  $t$ , defined as:

$$\phi(t) = b(t)p(t)a(t) - \gamma(t)x(t)$$

We now consider the following problem:

$$\text{Maximize: } \bar{\phi} \quad (21)$$

$$\text{Subject to: } 1) \quad \bar{Q} < \infty \quad (22)$$

$$2) \quad 0 \leq x(t) \leq x_{max} \forall t \quad (23)$$

$$3) \quad b(t) \in \{0, 1\}, 0 \leq p(t) \leq p_{max} \forall t \quad (24)$$

where  $\bar{\phi}$  is defined as the limiting time average profit:

$$\bar{\phi} \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\phi(\tau)\}$$

To solve the problem, we use the same queueing structure for  $Q(t)$  in (1) and the same virtual queue structure for  $Z(t)$  in (6), and use the same Lyapunov function  $L(\Theta(t))$  as defined before (recall that  $\Theta(t)$  is defined as the vector  $(Q(t), Z(t))$ ). However, we now consider the “penalty”  $-\phi(t)$ , and so the drift-plus-penalty technique seeks to choose a vector that minimizes a bound on:

$$\Delta(\Theta(t)) - V\mathbb{E}\{\phi(t)|\Theta(t)\}$$

Using the same analysis as Lemma 2, we can show the following bound on this drift-plus-penalty expression:

$$\begin{aligned} \Delta(\Theta(t)) - V\mathbb{E}\{\phi(t)|\Theta(t)\} &\leq B \\ &\quad - V\mathbb{E}\{b(t)p(t)F(p(t), y(t), \gamma(t)) - \gamma(t)x(t)|\Theta(t)\} \\ &\quad + Q(t)\mathbb{E}\{b(t)F(p(t), y(t), \gamma(t)) - s(t) - x(t)|\Theta(t)\} \\ &\quad + Z(t)\mathbb{E}\{\epsilon - s(t) - x(t)|\Theta(t)\} \end{aligned} \quad (25)$$

Our joint energy-allocation and pricing algorithm observes the current system state on each slot  $t$ , and chooses  $b(t)$ ,  $p(t)$ , and  $x(t)$  to minimize the right-hand side of the above drift expression (given the observed  $\Theta(t)$ ). This reduces to the following: Every slot  $t$ , observe queues  $Q(t)$ ,  $Z(t)$ , and observe  $s(t)$ ,  $\gamma(t)$ ,  $y(t)$ . Then choose a price  $p(t)$  and an allocation  $x(t)$  as follows:

- (Pricing  $p(t)$ ) Choose  $p(t)$  as the solution to:

$$\begin{aligned} \text{Max: } & F(p(t), y(t), \gamma(t))(Vp(t) - Q(t)) \\ \text{S.t.: } & 0 \leq p(t) \leq p_{max} \end{aligned}$$

If the resulting maximum value is non-negative, choose  $b(t) = 1$ . Else choose  $b(t) = 0$  so that no new requests are allowed on slot  $t$ .

- (Allocating  $x(t)$ ) Choose  $x(t)$  according to (12).
- (Queue Updates) Update  $Q(t)$  and  $Z(t)$  by (1) and (6).

This pricing policy does not need to know the demand state  $y(t)$  in the special case when  $F(p(t), y(t), \gamma(t)) = y(t)\hat{F}(p(t), \gamma(t))$ , so that demand state simply scales the demand function. This pricing structure is similar to that considered in [19] for wireless service providers.

### A. Defining Optimality

We define a  $(s, y, \gamma)$ -only policy as one that jointly chooses  $x^*(t)$ ,  $b^*(t)$ ,  $p^*(t)$  subject to  $0 \leq x^*(t) \leq x_{max}$ ,  $b^*(t) \in \{0, 1\}$ ,  $0 \leq p^*(t) \leq p_{max}$  according to a stationary and randomized decision that depends only on  $s(t)$ ,  $y(t)$ ,  $\gamma(t)$ . As in [11], it can be shown that the supremum time average profit  $\phi^*$  associated with the problem (21)-(24) can be achieved over the class of  $(s, y, \gamma)$ -only policies. Thus, there exists a  $(s, y, \gamma)$ -only policy  $x^*(t)$ ,  $b^*(t)$ ,  $p^*(t)$  that satisfies:

$$\mathbb{E} \{b^*(t)p^*(t)a^*(t) - \gamma(t)x^*(t)\} = \phi^* \quad (26)$$

$$\mathbb{E} \{a^*(t) - s(t) - x^*(t)\} \leq 0 \quad (27)$$

where  $a^*(t)$  represents the random requests on slot  $t$  associated with pricing decisions  $b^*(t)$ ,  $p^*(t)$  and under the random demand state  $y(t)$  and the random market price  $\gamma(t)$ . It is useful to define  $a^* \triangleq \mathbb{E} \{a^*(t)\}$ . In the case when the policy  $p^*(t)$ ,  $b^*(t)$ ,  $x^*(t)$  that satisfies (26)-(27) is not unique, we define  $a^*$  as the maximum value such that there exists an  $(s, y, \gamma)$ -only policy that satisfies (26)-(27).

### B. The Joint Pricing and Allocation Algorithm

**Theorem 2:** Assume that  $x_{max} \geq \max[a_{max}, \epsilon]$ , and that  $Q(0) = Z(0) = 0$ . If the above joint pricing and allocation policy is implemented every slot with fixed parameters  $\epsilon \geq 0$ ,  $V > 0$ , then:

a) The worst case delay  $D_{max}$  and backlog  $Q_{max}$  are the same as before (given in (15), (14)), where  $Q_{max}$  is proportional to  $V$  and  $D_{max}$  is proportional to  $V/\epsilon$ .

b) If the vector  $(s(t), y(t), \gamma(t))$  is i.i.d. over slots, and if  $\epsilon \leq \max[a^*, \mathbb{E} \{s(t)\}]$  (where  $a^*$  is defined in Section III-A), then:<sup>4</sup>

$$\frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E} \{\phi(\tau)\} \geq \phi^* - B/V \quad \forall t > 0$$

where  $B$  is defined in (11), and  $\phi^*$  is the optimal time average profit that can be achieved by any algorithm that satisfies the constraints of the problem (21)-(24).

*Proof:* See Appendix C. □

### IV. NON-I.I.D. MODELS

Here we extend the analysis to treat non-i.i.d. models. For brevity, we consider only the problem of Section II that seeks to allocate  $x(t)$  without regard to pricing.<sup>5</sup> Specifically, we assume that the processes  $s(t)$ ,  $a(t)$ ,  $\gamma(t)$  vary randomly over slots according to any probability model (with arbitrary time correlations). However, we continue to assume the sample paths are bounded so that  $0 \leq s(t) \leq s_{max}$ ,  $0 \leq a(t) \leq a_{max}$ ,  $0 \leq \gamma(t) \leq \gamma_{max}$  for all  $t$ . We show that the same algorithm of Section II, which allocates  $x(t)$  according to (12), still provides efficient performance in this context. We assume that

<sup>4</sup>Note that *actual profit* can be defined  $\tilde{\phi}(t) \triangleq b(t)p(t)a(t) - \gamma(t)\tilde{x}(t)$ , with  $\tilde{x}(t)$  defined in (13). Clearly  $\tilde{\phi}(t) \geq \phi(t)$  for all  $t$ , and so the time average of the actual profit  $\tilde{\phi}(t)$  is even closer to the optimal value  $\phi^*$ .

<sup>5</sup>Similar analysis can be applied to the pricing problem for this non-i.i.d. case, using the technique in [18] that incorporates the random demand  $a(t)$  with expectation  $F(p(t), y(t), \gamma(t))$ , where the  $y(t)$  and  $\gamma(t)$  processes are arbitrary sample paths.

$Q(0) = Z(0) = 0$ , and that fixed parameters  $V > 0$  and  $\epsilon \geq 0$  are used. We continue to assume that  $x_{max} \geq \max[a_{max}, \epsilon]$ .

We first observe that the exact same worst case backlog and delay bounds  $Q_{max}$  and  $D_{max}$  given in (14) and (15) hold in this non-i.i.d. case. Thus, worst case delay is still bounded by a constant that is proportional to  $V/\epsilon$ . This is because the proof of this bound in Theorem 1 (a) and (b) was a *sample path proof* that did not make use of the i.i.d. assumptions. Indeed, it used only the fact that  $0 \leq a(t) \leq a_{max}$  for all  $t$ .

It remains only to understand the efficiency of the time average cost. To this end, we use the  $T$ -slot lookahead metric as defined in the universal scheduling work [17][20]. Specifically, suppose that the sample path of  $(s(t), a(t), \gamma(t))$  is chosen at time 0 for all  $t$  according to some arbitrary values. For a given positive integer  $T$  and a positive integer  $R$ , we consider the first  $RT$  slots, composed of  $R$  successive “frames” of size  $T$ . For each frame  $r \in \{0, 1, \dots, R-1\}$ , we define  $c_r^*$  as the optimum solution to the following “ideal” problem that uses full knowledge of  $(s(t), a(t), \gamma(t))$  over the frame:

$$\text{Minimize: } c_r^* \triangleq \frac{1}{T} \sum_{\tau=rT}^{(r+1)T-1} \gamma(\tau)x(\tau) \quad (28)$$

$$\text{Subject to: 1) } \sum_{\tau=rT}^{(r+1)T-1} [s(\tau) + x(\tau) - a(\tau)] \geq 0 \quad (29)$$

$$2) \sum_{\tau=rT}^{(r+1)T-1} [s(\tau) + x(\tau) - \epsilon] \geq 0 \quad (30)$$

$$3) 0 \leq x(\tau) \leq x_{max} \forall \tau \in \{rT, \dots, (r+1)T-1\} \quad (31)$$

Thus,  $c_r^*$  is the optimal cost that can be achieved over frame  $r$ , considering all possible ways of allocating  $x(\tau)$  over this frame using perfect knowledge of the future values of  $(s(\tau), a(\tau), \gamma(\tau))$  over this frame, subject to ensuring the total energy provided over the frame is at least as much as the total sum arrivals, and is also at least  $\epsilon T$ .

**Theorem 3:** (Universal Scheduling) Under the above assumptions, the worst case backlog and delay are given by  $Q_{max}$  and  $D_{max}$  in (14) and (15). Further, for all positive integers  $T$  and  $R$ , we have:

$$\frac{1}{RT} \sum_{\tau=0}^{RT-1} \gamma(\tau)x(\tau) \leq \frac{1}{R} \sum_{r=0}^{R-1} c_r^* + \frac{BT}{V}$$

where  $B$  is defined in (11).

*Proof:* The proof combines the techniques of the proof of Theorem 1 with the universal scheduling results in [17][20], and is given in Appendix B. □

The above result says that the achieved time average cost over any interval of  $RT$  slots is less than or equal to the average of the  $c_r^*$  values, plus a “fudge factor” of at most  $BT/V$ . While the average of the  $c_r^*$  values is *not* the same as the minimum cost that could be achieved with perfect knowledge of the future over the full  $RT$  slots, this result is still interesting because the  $c_r^*$  values are still obtained by ideal algorithms implemented over  $T$  slot frames with full knowledge of the future events in these frames.

### V. EXPERIMENTAL EVALUATION

We evaluated the performance of the proposed algorithm on a six-month data set that we created by combining 10-minute average spot market prices  $\gamma(t)$  for Los Angeles area (LA1) from CAISO [21] and 10-minute energy production  $s(t)$  for a small subset of windfarms from the Western Wind resources

Dataset published by the National Renewable Energy Laboratory [22]. We modeled the demand  $a(t)$  as i.i.d. over slots and uniformly distributed over the integers  $\{0, 1, \dots, a_{max}\}$ . We executed the proposed Lyapunov drift optimization algorithm in 10-minute timeslots and experimented with different values of the parameters  $V, \epsilon$  and the corresponding deadlines they generate.

We compare the proposed algorithm against a simple greedy strategy ‘‘Purchase at deadline,’’ which tries to use all the available resource  $s(t)$  and only buys from the spot market as a last resort if a deadline is reached. As can be seen in Fig. 1, the proposed algorithm reduces the cost of the renewable supplier by approximately a factor of 2 in the tested six-month window. The slope of the two lines is different, suggesting that the savings are unbounded as the time increases. This is not surprising since the greedy strategy does not hedge for future high prices in the spot market while the proposed algorithm learns to proactively buy when the spot market prices are lower than typical and deadline violations seem probable. The high variability of the spot market prices [21] makes this advantage significant. The second observation, seen in Fig. 2, is that the proposed algorithm has on average a much smaller delay than the deadline, which for our parameters was  $D_{max} = 70$  hours. On the contrary, the greedy algorithm makes many requests wait close to (or exactly at) the maximum allowed 70 hours.

Our results use  $\epsilon = \mathbb{E}\{a(t)\} = a_{max}/2$ . We also conducted simulations with  $\epsilon = 0$ , which does not require knowledge of  $\mathbb{E}\{a(t)\}$ . While  $\epsilon = 0$  does not provide a finite delay guarantee, it still guarantees the same finite  $Q_{max}$ . Together with FIFO service, this means that the worst case delay for requests that arrive at time  $t$  is given by the smallest integer  $T > 0$  such that  $\sum_{\tau=t+1}^{t+T} s(\tau) \geq Q_{max}$ . While there is no bound on this for general  $s(t)$  processes, it can still lead to small delays. Indeed, in the simulations it *still* maintained all delays under  $D_{max} = 2.9$  days (having a maximum experimental delay of 14 hours, as compared to 9.5 hours for the  $\epsilon = \mathbb{E}\{a(t)\}$  case).<sup>6</sup> Fig. 1 shows it gives slightly better cost, particularly because it increases delay. Both Lyapunov optimization algorithms provided significantly better cost and delay as compared to the greedy algorithm. It should be noted that we did not compare against dynamic programming algorithms such as the one proposed in [3]. While it is clear that a dynamic programming approach could solve this problem optimally if the statistics of the underlying processes were known, one benefit of our approach is that no such prior knowledge is required. Further, the Lyapunov approach yields an efficient algorithm for multiple queues corresponding to different customers with different deadlines.

We now present some further experimental results investigating the influence of varying  $V$  and  $\epsilon$  in the performance of the proposed algorithm. For these simulations we used the same data set as the previous part. For the first experiment, the performance of the algorithm for different values of parameter  $V$  is compared. The rest of the parameters are unchanged and are  $a_{max} = 175$ ,  $\gamma_{max} = 180$ ,  $x_{max} = 400$ ,  $s_{max} = 90$ , and  $\epsilon = 87.5$ . The result is shown in Fig. 3. As expected, the cost

<sup>6</sup>For legibility, the delay data for the  $\epsilon = 0$  case is not shown in Fig. 2.

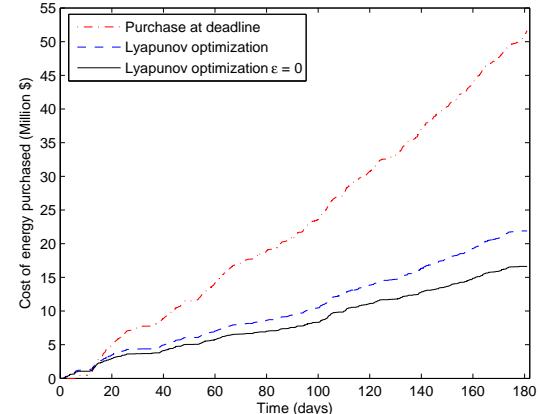


Fig. 1. Cost of the renewable energy supplier for energy purchased at the spot market. For the proposed algorithm we used the parameters  $a_{max} = 175$ ,  $\gamma_{max} = 180$ ,  $x_{max} = 400$ ,  $V = 100$ ,  $D_{max} = 415 = 2.9$  days.

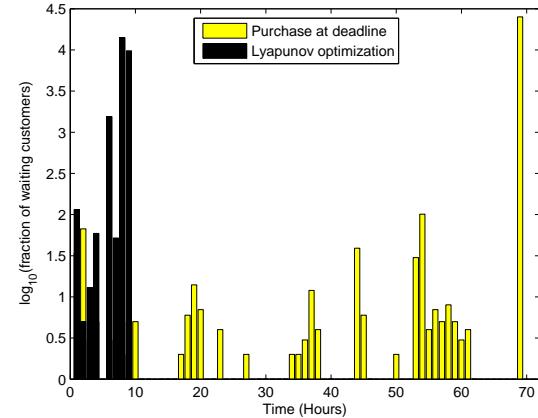


Fig. 2. Histogram of delay for the customers waiting in the service queues of the renewable energy supplier under the two algorithms (vertical axis in logarithmic scale). Case  $\epsilon = 0$  is not shown, but has max delay 14 hours, as compared with the  $\epsilon = \mathbb{E}\{A(t)\}$  case (shown) with max delay 9.5 hours. decreases with  $V$ . The tradeoff is in the maximum waiting time of the packets. The maximum waiting times observed in the simulations for parameter  $V$  being 20, 50, 100, 200 are 3.5, 5.8, 10.2, 15.2 hours, respectively.<sup>7</sup> For the second experiment, we consider the performance of our algorithm for different values of  $\epsilon$ . Here, we fixed the value  $V = 100$  and run the simulation for  $\epsilon = \{87.5, 60, 35, 10\}$ . The cost decreases as  $\epsilon$  decreases, as shown in Fig. 4. However, the maximum observed waiting times increase with  $\epsilon$ . So for  $\epsilon = 87.5, 60, 35, 10$ , the maximum observed waiting times are 9.5, 11.7, 12.5, 13.7 hours, respectively. Overall, as expected, the cost gets better as  $V$  is increased, with a tradeoff in waiting time. Further, the waiting time reduces as  $\epsilon$  increases to  $\mathbb{E}\{A(t)\}$ , although waiting times are still reasonable even with  $\epsilon = 0$ , which is useful when  $\mathbb{E}\{A(t)\}$  is unknown. For non-i.i.d. situations, using a smaller value of  $\epsilon$  may also reduce cost due to the fact that this relaxes the constraint (30).

## VI. CONCLUSIONS

This work presents a Lyapunov optimization approach to the problem of efficient use of renewable energy sources.

<sup>7</sup>The maximum observed waiting time for the simulation run for the  $V = 100$  case of Fig. 3 was 10.2, rather than 9.5 as in the previous simulation for the case  $V = 100$ . This is because this simulation used independently generated  $a(t)$  values.

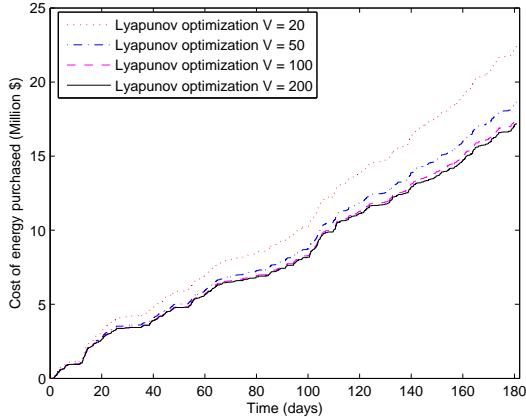


Fig. 3. Cost of the renewable energy supplier for energy purchased at the spot market for different values of  $V = 20, 50, 100, 200$ . For the proposed algorithm we used the parameters  $a_{max} = 175$ ,  $x_{max} = 400$ , and  $\epsilon = 87.5$ .

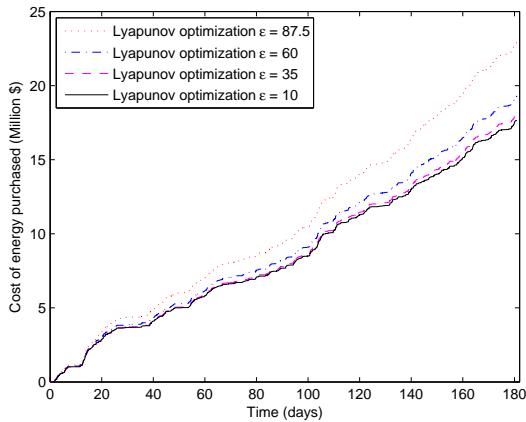


Fig. 4. Cost of the renewable energy supplier for energy purchased at the spot market for different values of  $\epsilon = 87.5, 60, 35, 10$ . For the proposed algorithm we used the parameters  $a_{max} = 175$ ,  $x_{max} = 400$ , and  $V = 100$ .

Efficiency can be improved if consumers are flexible and can tolerate their requests being served with some delay. Two different problems were presented: One that seeks to minimize cost associated with using an outside (possibly non-renewable) plant to meet the deadlines, and another that seeks to maximize profit by dynamically selecting a price for service. Our algorithms are simple and were shown to operate efficiently without knowing the statistical properties of the supply, demand, and energy request processes. We first considered a simple case when these processes are i.i.d. over slots but with unknown probabilities. We next treated the general case of arbitrary (possibly non-i.i.d. and non-ergodic) sample paths. Our analysis also contributes to the theory of Lyapunov optimization by introducing a new type of virtual queue that guarantees a bounded worst case delay. Our algorithms use a parameter  $V$  that can be tuned as desired to affect a performance-delay tradeoff, where achieved cost is within  $O(1/V)$  from optimal, with a worst case delay guarantee that is  $O(V)$ . These techniques provide a convenient alternative to dynamic programming that leads to a general framework for problems that naturally arise in scheduling of

renewable energy markets.

#### APPENDIX A – PROOF OF LEMMA 2

From the  $Z(t)$  update rule (6) we have:

$$Z(t+1) \leq \max[Z(t) - s(t) - x(t) + \epsilon, 0]$$

and hence:

$$Z(t+1)^2 \leq (Z(t) - s(t) - x(t) + \epsilon)^2$$

Thus:

$$\begin{aligned} \frac{Z(t+1)^2 - Z(t)^2}{2} &\leq \\ \frac{1}{2}(\epsilon - s(t) - x(t))^2 + Z(t)(\epsilon - s(t) - x(t)) & \\ \leq \frac{1}{2} \max[(s_{max} + x_{max})^2, \epsilon^2] + Z(t)(\epsilon - s(t) - x(t)) & \end{aligned}$$

Similarly, by squaring (1) and using the inequality:

$$(\max[Q - \mu, 0] + a)^2 \leq Q^2 + \mu^2 + a^2 + 2Q(a - \mu)$$

which holds for any  $Q \geq 0$ ,  $\mu \geq 0$ ,  $a \geq 0$ , we obtain:

$$\begin{aligned} \frac{Q(t+1)^2 - Q(t)^2}{2} &\leq \frac{1}{2}[(s_{max} + x_{max})^2 + a_{max}^2] \\ &+ Q(t)(a(t) - s(t) - x(t)) \quad (32) \end{aligned}$$

Combining the above yields:

$$\begin{aligned} L(\Theta(t+1)) - L(\Theta(t)) &\leq B \\ &+ Q(t)(a(t) - s(t) - x(t)) + Z(t)(\epsilon - s(t) - x(t)) \quad (33) \end{aligned}$$

Taking conditional expectations of the above, given  $\Theta(t)$ , and adding  $V\mathbb{E}\{\gamma(t)x(t)|\Theta(t)\}$  to both sides proves the result.

#### APPENDIX B – PROOF OF THEOREM 3

Again define  $\Theta(t) \triangleq [Q(t), Z(t)]$ , and define the Lyapunov function  $L(\Theta(t))$  the same as before:

$$L(\Theta(t)) \triangleq \frac{1}{2}[Q(t)^2 + Z(t)^2]$$

As in [20][17], for a given integer  $T > 0$ , we define the  $T$ -slot sample path drift  $\Delta_T(\Theta(t))$  as follows:

$$\Delta_T(\Theta(t)) \triangleq L(\Theta(t+T)) - L(\Theta(t))$$

This differs from our 1-slot conditional drift  $\Delta(\Theta(t))$ , used for the i.i.d. analysis, because (i) It involves  $T$  slots, rather than 1 slot, and (ii) It does not use an expectation.

Now suppose that the values  $(a(\tau), s(\tau), \gamma(\tau))$  and  $x(\tau)$  satisfy the following for all  $\tau$ :

$$0 \leq a(\tau) \leq a_{max}, \quad 0 \leq s(\tau) \leq s_{max} \quad (34)$$

$$0 \leq \gamma(\tau) \leq \gamma_{max}, \quad 0 \leq x(\tau) \leq x_{max} \quad (35)$$

We have the following lemma.

**Lemma 4:** Fix any slot  $t$ , any queue state  $\Theta(t) = [Q(t), Z(t)]$ , and any integer  $T > 0$ . Consider an arbitrary sample path for  $a(\tau), s(\tau), \gamma(\tau)$ , over the interval  $\tau \in \{t, t+1, \dots, t+T-1\}$ , assumed only to satisfy (34)-(35). Assume that the decisions for  $x(\tau)$  are given by the algorithm (12),

with queue updates for  $Q(t)$  and  $Z(t)$  given by (1) and (6). Then:

$$\begin{aligned} \Delta_T(\Theta(t)) + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) &\leq \\ BT^2 + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x^*(\tau) \\ + Q(t) \sum_{\tau=t}^{t+T-1} [a(\tau) - s(\tau) - x^*(\tau)] \\ + Z(t) \sum_{\tau=t}^{t-1} [\epsilon - s(\tau) - x^*(\tau)] \end{aligned}$$

where  $x^*(\tau)$  are any alternative choices that satisfy  $0 \leq x^*(\tau) \leq x_{max}$  for all  $\tau \in \{t, \dots, t+T-1\}$ . The constant  $B$  is given in (11).

*Proof:* From (33) we have that for all  $\tau$ :

$$\begin{aligned} L(\Theta(\tau+1)) - L(\Theta(\tau)) &\leq B + Q(\tau)(a(\tau) - s(\tau) - x(\tau)) \\ &\quad + Z(\tau)(\epsilon - s(\tau) - x(\tau)) \end{aligned}$$

Summing the result over  $\tau \in \{t, \dots, t+T-1\}$  yields:

$$\begin{aligned} \Delta_T(\Theta(t)) &\leq BT + \sum_{\tau=t}^{t+T-1} Q(\tau)(a(\tau) - s(\tau) - x(\tau)) \\ &\quad + \sum_{\tau=t}^{t+T-1} Z(\tau)(\epsilon - s(\tau) - x(\tau)) \end{aligned}$$

Adding the penalty term to both sides yields:

$$\begin{aligned} \Delta_T(\Theta(t)) + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) &\leq BT + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) \\ &\quad + \sum_{\tau=t}^{t+T-1} Q(\tau)(a(\tau) - s(\tau) - x(\tau)) \\ &\quad + \sum_{\tau=t}^{t+T-1} Z(\tau)(\epsilon - s(\tau) - x(\tau)) \end{aligned}$$

We now use the fact that for each slot  $\tau$ , the value of  $x(\tau)$  is chosen to minimize:

$$x(\tau)[V\gamma(\tau) - Q(\tau) - Z(\tau)]$$

over all  $x(\tau)$  such that  $0 \leq x(\tau) \leq x_{max}$ . It follows that:

$$\begin{aligned} \Delta_T(\Theta(t)) + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) &\leq BT + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x^*(\tau) \\ &\quad + \sum_{\tau=t}^{t+T-1} Q(\tau)(a(\tau) - s(\tau) - x^*(\tau)) \\ &\quad + \sum_{\tau=t}^{t+T-1} Z(\tau)(\epsilon - s(\tau) - x^*(\tau)) \end{aligned}$$

where for all  $\tau \in \{t, \dots, t+T-1\}$ ,  $x^*(\tau)$  is any value that satisfies  $0 \leq x^*(\tau) \leq x_{max}$ . Now note that the maximum changes in the  $Q(\tau)$  and  $Z(\tau)$  queues on one slot are given by constants  $C_Q$  and  $C_Z$ , respectively, defined:

$$\begin{aligned} C_Q &\triangleq \max[s_{max} + x_{max}, a_{max}] \\ C_Z &\triangleq \max[s_{max} + x_{max}, \epsilon] \end{aligned}$$

Thus:

$$\begin{aligned} |(Q(\tau) - Q(t))(a(\tau) - s(\tau) - x^*(\tau))| &\leq C_Q^2(\tau - t) \\ |(Z(\tau) - Z(t))(\epsilon - s(\tau) - x^*(\tau))| &\leq C_Z^2(\tau - t) \end{aligned}$$

We can thus replace the right hand side of the above drift inequality with:

$$\begin{aligned} \Delta_T(\Theta(t)) + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x(\tau) &\leq \\ BT + \frac{(C_Q^2 + C_Z^2)T(T-1)}{2} \\ + V \sum_{\tau=t}^{t+T-1} \gamma(\tau)x^*(\tau) + Q(t) \sum_{\tau=t}^{t+T-1} (a(\tau) - s(\tau) - x^*(\tau)) \\ + Z(t) \sum_{\tau=t}^{t+T-1} (\epsilon - s(\tau) - x^*(\tau)) \end{aligned}$$

where we have used the fact that  $\sum_{\tau=t}^{t+T-1}(\tau - t) = T(T-1)/2$ . However, it is not difficult to show that:

$$\frac{(C_Q^2 + C_Z^2)}{2} \leq B$$

and hence:

$$BT + \frac{(C_Q^2 + C_Z^2)}{2}T(T-1) \leq BT^2$$

This proves the result.  $\square$

Now fix a frame size  $T > 0$ , consider the timeline decomposed into  $R$  successive frames of size  $T$ , and consider any frame  $r \in \{0, 1, \dots, R-1\}$ . Define  $c_r^*$  as the optimum cost in the frame- $r$  problem (28)-(31), and define  $x^*(\tau)$  for  $\tau \in \{rT, \dots, rT+T-1\}$  as the optimal decisions for that problem, which achieve  $c_r^*$  and satisfy the inequality constraints (29)-(31). Then using the drift bound given in Lemma 4 together with the equalities and inequalities (28)-(31), we have:

$$\Delta_T(\Theta(rT)) + V \sum_{\tau=rT}^{rT+T-1} \gamma(\tau)x(\tau) \leq BT^2 + VTc_r^*$$

Summing the above over  $r \in \{0, 1, \dots, R-1\}$ , using the definition of  $\Delta_T(\Theta(t))$ , and dividing by  $RTV$  yields:

$$\begin{aligned} \frac{L(\Theta(RT)) - L(\Theta(0))}{RTV} + \frac{1}{RT} \sum_{\tau=0}^{RT-1} \gamma(\tau)x(\tau) &\leq \\ \frac{BT}{V} + \frac{1}{R} \sum_{r=0}^{R-1} c_r^* & \end{aligned}$$

Using the fact that  $L(\Theta(0)) = 0$  and  $L(\Theta(RT)) \geq 0$  yields the result.

## APPENDIX C – PROOF OF THEOREM 2

Part (a) follows by noting that the proof of parts (a) and (b) in Theorem 1 hold exactly in this new context, as we have not changed the queueing dynamics for  $Q(t)$  or  $Z(t)$  or the fact that  $a(t) \leq a_{max}$  for all  $t$ .

We now prove part (b). We have assumed that  $\epsilon \leq \max[a^*, \mathbb{E}\{s(t)\}]$ . We first prove the result for the case  $\epsilon \leq$

$a^*$ . On each slot  $t$  our dynamic algorithm makes actions  $b(t)$ ,  $p(t)$ ,  $x(t)$  that, given the observed  $\Theta(t) = [Q(t), Z(t)]$ , minimizes the right hand side of the drift inequality (25) over all alternative choices. Thus:

$$\begin{aligned} \Delta(\Theta(t)) - V\mathbb{E}\{\phi(t)|\Theta(t)\} &\leq B \\ -V\mathbb{E}\{b^*(t)p(t)F(p^*(t), y(t), \gamma(t)) - \gamma(t)x^*(t)|\Theta(t)\} \\ +Q(t)\mathbb{E}\{b^*(t)F(p^*(t), y(t), \gamma(t)) - s(t) - x^*(t)|\Theta(t)\} \\ +Z(t)\mathbb{E}\{\epsilon - s(t) - x^*(t)|\Theta(t)\} \end{aligned} \quad (36)$$

where  $b^*(t)$ ,  $p^*(t)$ ,  $x^*(t)$  are any other choices that satisfy:

$$0 \leq x^*(t) \leq x_{max}, 0 \leq p^*(t) \leq p_{max}, b^*(t) \in \{0, 1\} \forall t$$

We now use the existence of a  $(s, y, \gamma)$ -only policy  $x^*(t)$ ,  $b^*(t)$ ,  $p^*(t)$  that satisfies the inequalities (26)-(27). It is not difficult to show that (26)-(27) are equivalent to the following:

$$\mathbb{E}\{b^*(t)p^*(t)F(p^*(t), y(t), \gamma(t)) - \gamma(t)x^*(t)|\Theta(t)\} = \phi^* \quad (37)$$

$$\mathbb{E}\{b^*(t)F(p^*(t), y(t), \gamma(t)) - s(t) - x^*(t)|\Theta(t)\} \leq 0 \quad (38)$$

$$\mathbb{E}\{b^*(t)F(p^*(t), y(t), \gamma(t))|\Theta(t)\} = a^* \quad (39)$$

where the above conditional expectations (37)-(39) given  $\Theta(t)$  are the same as the unconditional expectations, because the  $(s, y, \gamma)$ -only policy does not depend on the queue states  $\Theta(t)$  (recall that  $(s(t), y(t), \gamma(t))$  is i.i.d. over slots and hence independent of queue states). Plugging (37)-(39) directly into the right hand side of (36) yields:

$$\Delta(\Theta(t)) - V\mathbb{E}\{\phi(t)|\Theta(t)\} \leq B - V\phi^* + Z(t)(\epsilon - a^*) \quad (40)$$

Because we have assumed that  $\epsilon \leq a^*$ , this reduces to:

$$\Delta(\Theta(t)) - V\mathbb{E}\{\phi(t)|\Theta(t)\} \leq B - V\phi^* \quad (41)$$

Taking expectations of the above (with respect to the random  $\Theta(t)$ ) and using the law of iterated expectations gives:

$$\mathbb{E}\{L(\Theta(t+1))\} - \mathbb{E}\{L(\Theta(t))\} - V\mathbb{E}\{\phi(t)\} \leq B - V\phi^*$$

The above holds for all slots  $t$ . Summing over  $\tau \in \{0, \dots, M-1\}$  for some integer  $M > 0$  yields:

$$\begin{aligned} \mathbb{E}\{L(\Theta(M))\} - \mathbb{E}\{L(\Theta(0))\} - V \sum_{\tau=0}^{M-1} \mathbb{E}\{\phi(\tau)\} &\leq \\ M(B - V\phi^*) \end{aligned}$$

Dividing by  $VM$  and using the fact that  $\mathbb{E}\{L(\Theta(0))\} = 0$  and  $\mathbb{E}\{L(\Theta(M))\} \geq 0$  yields:

$$-\frac{1}{M} \sum_{\tau=0}^{M-1} \mathbb{E}\{\phi(\tau)\} \leq -\phi^* + B/V$$

This holds for all  $M > 0$ , proving the result for the case  $\epsilon \leq a^*$ .

We have used the fact that  $\epsilon \leq a^*$  only in showing the  $Z(t)(\epsilon - a^*)$  term on the right hand side of (40) can be removed while preserving the inequality. However, suppose that  $\epsilon \leq \mathbb{E}\{s(t)\}$ . Then the  $Z(t)\mathbb{E}\{\epsilon - s(t) - x^*(t)|\Theta(t)\}$  term in the right hand side of (36) can immediately be removed (recall that  $x^*(t) \geq 0$  and  $\mathbb{E}\{s(t)\} = \mathbb{E}\{s(t)|\Theta(t)\}$  because  $s(t)$  is i.i.d. over slots and hence independent of current queue backlog). This leads directly to (41) regardless of the value of  $a^*$ . Thus, the result holds whenever  $\epsilon \leq \max[a^*, \mathbb{E}\{s(t)\}]$ , proving the theorem.

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